Biom. J. 35 (1993) 1, 29–45 Akademie Verlag

Estimating Population Size for Continuous-Time Capture-Recapture Models Via Sample Coverage

Anne Chao
National Tsing Hua University, Taiwan

S.-M. Lee
Feng-Chia University, Taiwan

Summary

An estimation procedure using the idea of sample coverage is proposed to estimate population size for capture-recapture experiments in continuous time. The capture rates (intensity) are allowed to vary by time and individuals (heterogeneity). Only capture frequency history are sufficient for estimating population size while capture times and sequential orders of animals caught are irrelevant for the analysis. An example is given for illustration. The performance of the proposed estimation procedure is also investigated by simulation.

Key words: Sample coverage; Population size; Coefficient of variation; Poisson process.

1. Introduction

Consider a closed population with size N and the animals are indexed by $1, 2, \ldots, N$ in any arbitrary fashion. In a usual t-sample capture-recapture experiment, animals are captured and marked in each trapping sample (or trapping occasion), and we only notice in each sample whether each animal was captured or not without recording the sequential order or the time when it was captured. Thus the capture history can be expressed as a $N \times t$ matrix (A_{ij}), where $A_{ij} = I$ [the ith animals was caught in the jth sample], $i = 1, 2, \ldots, N, j = 1, 2 \ldots, t$. Extensive models have been discussed for the above discrete-time type data, see Seber (1982, 1986) and Otis et al. (1978) for a review.

If the capture time and the sequential order for each animal can be recorded, then a continuous-time model is needed. Recently, Becker (1984), Yip (1989), Anderson and Wilson (1989) and Becker and Heyde (1990) have addressed this problem. Becker (1984), Becker and Heyde (1990) proposed an estimation procedure using a martingale approach. Anderson and Wilson (1989) have done some simulation to compare the existing estimators for continuous-time models. Some of the previous results will be briefly reviewed in Section 2.

Chao, Lee and Jeng (1992), Lee and Chao (1992) used the idea of sample coverage to derive a unified estimation procedure for the sequence models of discrete time proposed by Otis et al. (1978). In this paper, we use exactly the same idea to present an estimation procedure for continuous-time models in Section 2. A cottontail rabbit example from EDWARDS and EBERHARDT (1967) is provided in Section 3 for illustration. A limited simulation study is reported in Section 4 to show the general performance of the proposed estimators and to compare it with previous methods derived from a martingale approach.

2. Sample Coverage and Estimation

Let $X_i(t)$ be the number of times that the animal i has been caught in $[0, t]$. $X_i(t)$ is a continuous time counting process with right continuous paths and intensity function $\lambda_i(t)$

$$\lambda_i(t) = \lim_{h \to 0^+} h^{-1} P[X_i(t+h) - X_i(t) = 1 \mid \mathfrak{F}_t],$$

where $\mathfrak{F}_t$ denotes the σ-field generated by $\{X_1(u), X_2(u), \ldots, X_N(u); u \leqq t\}$. We assume that the intensity function can vary with time and individual, that is, $\lambda_i(t) = \alpha_i \beta(t)$, $i = 1, 2, \ldots, N$, where α_i denotes the unknown individual effect of the ith animal and $\beta(t)$ denotes the unknown time effect on the capture rate. This is a continuous Model M_{th} (see OTIS et al. 1978 for the definition of various models). If all α_i's are equal, it reduces to a continuous Model M_t; if $\beta(t)$ is a constant, it reduces to a continuous Model M_h.

We first briefly review BECKER's (1984) estimator for Model M_t: Let Z_t be the number of captures and D_t be the number of distinct animals that are caught by time t, i.e., $Z_t = \sum_i X_i(t)$, $D_t = \sum_i I[X_i(t) > 0]$. It follows from AALEN (1978) that $NK_t - \int D_{U-} dZ_U$ is a zero mean martingale, where $K_t = Z_t - D_t$. Thus an estimator of N is obtained by equating this martingale to 0:

$$\hat{N}_t = \int D_{U-} dZ_U / K_t. \tag{2.1}$$

It is easy to show that this estimator is equivalent to SCHNABEL (1938) estimator if each capture is regarded as a trapping sample. BECKER (1984) also obtained a variance estimator for $\hat{N}_t$. For Model M_{th}, let $Y_i(t)$ be the number of animals which have been caught at least i times by time t. He proposed the following estimator for Model M_{th}:

$$\begin{aligned}\hat{N}_{th} = {} & [Y_1 + 2 \int Y_1 (Y_1 - Y_2)^{-1} dY_2 - \int Y_1 (Y_2 - Y_3)^{-1} dY_3] \\ & [2 \int (Y_1 - Y_2)^{-1} dY_2 - \int (Y_2 - Y_3)^{-1} dY_3]^{-1},\end{aligned} \tag{2.2}$$

where Y_i appearing in the integrand is adjusted to be left continuous. A variance estimator for $\hat{N}_{th}$ is also provided in Becker's paper.

The maximum likelihood estimator (MLE) $\hat{N}_{mle}$ under Model M_t derived by BECKER and HEYDE (1990) satisfies that

$$Z_t/\hat{N}_{mle} = \int (\hat{N}_{mle} - D_{U-})^{-1} \, dD_U \tag{2.3}$$

with a variance estimator

$$(\hat{N}_{mle})^2 \left[\int_0^t (\hat{N}_{mle} - D_{U-})^{-1} D_{U-} \, dZ_U \right]^{-1} \tag{2.4}$$

An alternative variance estimator which will be used in the simulation comparisons of Section 4 was given in DARROCH (1958):

$$\hat{N}_{mle} \left[\int_0^t (\hat{N}_{mle} - D_{U-})^{-2} D_{U-} \, dD_U \right]^{-1}. \tag{2.5}$$

BECKER and HEYDE (1990) also introduced a class of simple estimators including $\hat{N}_t$ with good asymptotic efficiency relative to the MLE, but only $\hat{N}_{mle}$ and $\hat{N}_t$ will be compared here.

We now describe our estimation procedure. Each individual parameter α_i is not of interest and the essentially relevant parameters are the mean $\bar{\alpha} = \sum_i \alpha_i/N$ and coefficient of variation (CV) $\gamma = [\sum_i (\alpha_i - \bar{\alpha})^2/N]^{1/2}/\bar{\alpha}$. ($\gamma = 0$ is equivalent to that all α_i's are equal). Also we will only concentrate on the proportion of the total individual effects of captured animals, the sample coverage for time $[0, t]$, which is defined as

$$C_t = \sum_{i=1}^{N} \alpha_i \, I[X_i(t) > 0] / \sum_{i=1}^{N} \alpha_i. \tag{2.6}$$

In the literature, the sample coverage of a random sample is originally discussed for a multinomial population and is defined to be the sum of the probabilities of the observed classes. The previous references include GOOD (1953), GOOD & TOULMIN (1956), ROBBINS (1968), ENGEN (1978), ESTY (1982, 1983, 1985, 1986).

For Model $M_t(\gamma = 0)$, C_t reduces to D_t/N. Hence a natural estimator of N in this case is

$$\hat{N}_0 = D_t/\hat{C}_t, \tag{2.7}$$

where $\hat{C}_t$ will be derived in (2.16). This estimator was first introduced by DARROCH & RATCLIFF (1980) for a multinomial model. They found that $\hat{N}_0$ is asymptotically remarkably efficient in the equally-likely multinomial cases when compared with the usual MLE.

When α_i's are different ($\gamma > 0$), we first evaluate the difference between $E(D_t)/E(C_t)$ and N. Define

$$\tau = \int_0^t \beta(u)\, du, \tag{2.8}$$

then

$$E(D_t) = N - \sum_1^N \exp(-\alpha_i \tau), \tag{2.9}$$

and

$$E(C_t) = 1 - \sum_1^N \alpha_i \exp(-\alpha_i \tau) / \sum_1^N \alpha_i, \tag{2.10}$$

we can write that

$$E(D_t)/E(C_t) = N + g(\alpha)/[N\bar{\alpha} E(C_t)],$$

where

$$g(\boldsymbol{\alpha}) = N \sum_{i=1}^N \alpha_i \exp(-\alpha_i \tau) - \left(\sum_1^N \alpha_i\right)\left[\sum_1^N \exp(-\alpha_i \tau)\right].$$

Expanding $g(\boldsymbol{\alpha})$ at $\bar{\boldsymbol{\alpha}} = (\bar{\alpha}, \bar{\alpha}, \ldots, \bar{\alpha})$ to the second order term and noting that

$$g(\bar{\boldsymbol{\alpha}}) = 0;$$

$$\left[\frac{\partial g(\boldsymbol{\alpha})}{\partial \alpha_i}\bigg|_{\boldsymbol{\alpha} = \bar{\boldsymbol{\alpha}}}\right] = 0 \quad \text{for all } i = 1, 2, \ldots, N;$$

$$\frac{\partial^3 g(\boldsymbol{\alpha})}{\partial \alpha_i \partial \alpha_j \partial \alpha_k} = 0, \quad \text{for } i, j, k \text{ different.}$$

Thus (all indexes in the following proof are running from 1 to N)

$$g(\boldsymbol{\alpha}) = \frac{1}{2} \sum_i \left[\frac{\partial^2 g(\boldsymbol{\alpha})}{\partial \alpha_i^2}\bigg|_{\boldsymbol{\alpha} = \bar{\boldsymbol{\alpha}}}\right] (\alpha_i - \bar{\alpha})^2 + \frac{1}{2} \sum_{i \neq j}\sum \left[\frac{\partial^2 g(\boldsymbol{\alpha})}{\partial \alpha_i \partial \alpha_j}\bigg|_{\boldsymbol{\alpha} = \bar{\boldsymbol{\alpha}}}\right] (\alpha_i - \bar{\alpha})(\alpha_j - \bar{\alpha}) + R_1,$$

where R_1 denotes the remainder term and

$$R_1 = \frac{1}{6} \sum_i \left[\frac{\partial^3 g(\boldsymbol{\alpha})}{\partial \alpha_i^3}\bigg|_{\boldsymbol{\alpha} = \boldsymbol{\alpha}^*}\right] (\alpha_i - \bar{\alpha})^3 + \frac{1}{2} \sum_{i \neq j}\sum \left[\frac{\partial^3 g(\boldsymbol{\alpha})}{\partial \alpha_i^2 \partial \alpha_j}\bigg|_{\boldsymbol{\alpha} = \boldsymbol{\alpha}^*}\right] (\alpha_i - \bar{\alpha})^2 (\alpha_j - \bar{\alpha}),$$

and $\boldsymbol{\alpha}^* = (\alpha_1^*, \alpha_2^*, \ldots, \alpha_N^*)$, $\alpha_i^* = \bar{\alpha} + \theta(\alpha_i - \bar{\alpha})$, $i = 1, 2, \ldots, N$, for some $0 < \theta < 1$. Substituting all the derivatives and using that $\sum_1^N \alpha_i = \sum_1^N \alpha_i^* = N\bar{\alpha}$, $\sum_{j \neq i}^N (\alpha_j - \bar{\alpha}) = \bar{\alpha} - \alpha_i$, we then obtain that

$$\frac{E(D_t)}{E(C_t)} = N - \frac{N\tau\bar{\alpha}\, e^{-\tau\bar{\alpha}}}{E(C_t)} \gamma^2 + R_2, \tag{2.11}$$

where

$$R_2 = \frac{1}{2E(C_t)} \tau^2 \bar{\alpha}^2 \sum_1^N e^{-\tau\alpha_i^*} \left(\frac{\alpha_i - \bar{\alpha}}{\bar{\alpha}}\right)^3 - \frac{\theta}{6E(C_t)} \tau^3 \bar{\alpha}^3 \sum_1^N e^{-\tau\alpha_i^*} \left(\frac{\alpha_i - \bar{\alpha}}{\bar{\alpha}}\right)^4.$$

Define $f_k(t) = \sum I[X_i(t) = k]$ be the number of animals captured exactly k times by time t. Note that

$$E(f_1(t)) = \sum_i \tau\alpha_i \exp(-\tau\alpha_i) \tag{2.12}$$

$$= N\tau\bar{\alpha} \exp(-\tau\bar{\alpha}) + R_3, \tag{2.13}$$

where

$$R_3 = \frac{N}{2} \tau^2 \bar{\alpha}^2 (\tau\bar{\alpha} - 2)\, \mathrm{e}^{-\tau\bar{\alpha}} \gamma^2 + \frac{1}{6} \tau^3 \bar{\alpha}^3 \sum_1^N (3 - \tau\alpha_i^{**})\, e^{-\tau\alpha_i^{**}} \left(\frac{\alpha_i - \bar{\alpha}}{\bar{\alpha}}\right)^3,$$

$\alpha_i^{**} = \bar{\alpha} + \theta^*(\alpha_i - \bar{\alpha})$, $i = 1, 2, \ldots, N$, for some $0 < \theta^* < 1$. It follows from (2.11) and (2.13) that

$$N = \frac{E(D_t)}{E(C_t)} + \frac{E(f_1(t))}{E(C_t)} \gamma^2 - R, \tag{2.14}$$

where $R = R_2 + R_3\gamma^2/E(C_t)$. Extensive numerical results have suggested that R is usually negligible. Therefore our main result will be based on the following approximation:

$$N \approx \frac{E(D_t)}{E(C_t)} + \frac{E(f_1(t))}{E(C_t)} \gamma^2. \tag{2.15}$$

We first discuss the estimation of $E(C_t)$. From (2.10), (2.12) and $E(\sum X_i(t)) = \sum \alpha_i \tau$, an estimator for $E(C_t)$ clearly is

$$\hat{C}_t = 1 - f_1(t)/\sum_i X_i(t) = 1 - f_1(t)/\sum_i i f_i(t). \tag{2.16}$$

We now estimate CV. Since $E[\sum_i X_i(t)(X_i(t)-1)]=\sum_i i(i-1)E(f_i(t))=\tau^2\sum\alpha_i^2$, we have

$$\gamma^2=N\sum\alpha_i^2/(\sum\alpha_i)^2-1=N\sum i(i-1)E(f_i(t))/(E\sum if_i(t))^2-1.$$

In order to obtain an estimator of γ^2, we must replace N in the above by an initial estimate. From (2.7), we consider $\hat{N}_0$ as an initial estimate, thus we have the following estimator of the nonnegative parameter γ^2:

$$\hat{\gamma}^2=\max\{\hat{N}_0\sum i(i-1)f_i(t)/[\sum if_i(t)]^2-1,0\}. \tag{2.17}$$

From (2.15), we then propose the following estimator for CV > 0:

$$\hat{N}_1=\frac{D_t}{\hat{C}_t}+\frac{f_1(t)}{\hat{C}_t}\hat{\gamma}^2. \tag{2.18}$$

For the variance estimator of the proposed estimator, we assume that $\alpha_1, \alpha_2, \ldots, \alpha_N$ are a random sample from an unknown distribution $F(\alpha)$, then unconditionally, $(f_0(t), f_1(t), \ldots, f_m(t))$, m is the maximum frequency, is approximately multinomially distributed with parameter N and cell probability $\int(\tau\alpha)^i\exp(-\tau\alpha)/i!\,dF(\alpha)$, $i=0,1,\ldots,m$, where τ is defined in (2.8). Notice that both $\hat{N}_0$ and $\hat{N}_1$ are functions of $(f_1(t), f_2(t), \ldots, f_m(t))$, thus the asymptotic normality follows directly. Variance estimators can be obtained as follows: for example, for $\hat{N}_1$ if $\hat{\gamma}^2>0$, (dropping the variable t of $f_i(t)$ in the following)

$$\hat{N}_1=\frac{\sum f_i}{1-f_1/\sum if_i}+\frac{f_1}{1-f_1/\sum if_i}\left[\frac{\sum f_i}{1-f_1/\sum if_i}\frac{\sum i(i-1)f_i}{(\sum if_i)^2}-1\right],$$

which then implies

$$\hat{\text{var}}(\hat{N}_1)\approx\sum_{i=1}^{m}\sum_{j=1}^{m}\frac{\partial\hat{N}_1}{\partial f_i}\frac{\partial\hat{N}_1}{\partial f_j}\hat{\text{cov}}(f_i,f_j), \tag{2.19}$$

where

$$\hat{\text{cov}}(f_i,f_j)=\begin{cases}f_i(1-f_i/\hat{N}_1) & \text{if } i=j\\ -f_if_j/\hat{N}_1 & \text{if } i\neq j\end{cases}.$$

The adequacy of this variance estimator will be numerically checked in the simulation study of Section 4.

Remark 1: The estimators proposed in BECKER (1984) and BECKER and HEYDE (1990) as well as the estimators presented in this work are all independent of the capture times. In other words, the exact capture times are not necessary for the analysis. This is easily seen from the likelihood described below: Let $t_{i,1}, t_{i,2}, \ldots, t_{i,X_i(t)}$ be the capture times for the ith animal, where $X_i(t)$ is the number of times that the animal i has been caught by time t (defined in the beginning of this section). Under the assumption that the intensity function $\lambda_i(t) = \alpha_i \beta(t)$, the likelihood function by time t is

$$\left(\prod_{i=1}^{N} \alpha_i^{X_i(t)}\right)\left(\prod_{X_i(t)>0} \prod_{j=1}^{X_i(t)} \beta(t_{i,j})\right) \exp\left[-\left(\sum_{1}^{N} \alpha_i\right) \int_0^t \beta(u)\, du\right]. \quad (2.20)$$

Thus the exact capture times $\{t_{i,j}\}$ are relevant only in estimating the function $\beta(u)$. As far as the estimation of N is concerned, the sufficient statistics are the capture frequency history $\{X_i(t), i=1, 2, \ldots, N\}$. The capture frequency count $f_i t)$ is a function of the sufficient statistics. If we further assume that $\alpha_1, \alpha_2, \ldots, \alpha_N$ are a random sample from an unknown distribution $F(\alpha)$, then the sufficient statistics become $\{f_i(t), i=1, 2, \ldots, m\}$ with respect to the unconditional density.

Remark 2: The MLE derived in BECKER and HEYDE (1990), our estimator $\hat{N}_0$ (Eq. 2.7) for Model M_t and $\hat{N}_1$ (Eq. 2.18) for Model M_{th} are also independent of the sequential order (recall that our estimators are functions of f_i's only). Other estimators derived from a martingale approach, however, do depend on the sequential order of the captures. For example, if the sequence of capture order is

1, 1, 1, 2, 1, 1, 2, 1, 1, 2, 3, 4,

then $\hat{N}_t = 2.5$ and $\hat{N}_{th} = 3.5$. Now suppose the sequential order changes to

1, 2, 3, 4, 1, 1, 1, 1, 1, 1, 2, 2,

then $\hat{N}_t = 4.75$ and $\hat{N}_{th} = -0.8$. For both orders, the proposed $\hat{N}_0 = 4.8$, $\hat{N}_1 = 6.24$ and the solution of (2.3) for MLE is 3.48. The first example shows that $\hat{N}_t$ and $\hat{N}_{th}$ can be less than the number of observed animals, which is 4; the second example shows that $\hat{N}_{th}$ can yield negative estimates, as found by ANDERSON and WILSON (1989). The proposed estimators are clearly free of these drawbacks.

3. An Example

Since our analysis depends only on the capture frequency, any data set for discrete-time can be analyzed by using a continuous-time model. We will consider the data set provided in EDWARDS and EBERHARDT (1967) with known population size. In their study, 135 wild cottontail rabbits were penned in a rabbit-proof enclosure. Live trapping was conducted for 18 consecutive nights. Out of 142 captures, 'there $D_t = 76$ distinct rabbits. The ordered capture frequencies (f_1 to f_7)were 43, 16, 8, 6 0, 2, 1. This data set has been studied in BURNHAM and OVERTON (1978), OTIS et al. (1978), CHAO (1987), CHAO et al. (1992) using a discrete-time model. Although this is a discrete-time experiment, we can analyze it based on a continuous model as if the animals were sequentially captured. First, the MLE for continuous Model M_t is 99.5 with s.e. 7.6, which is generally consistent with those for discrete Model M_t (e.g., Darroch's MLE is 96). For our analysis, the sample coverage estimate from (2.16) is $1-43/142=69.72\%$. Hence our estimate for a continuous Model M_t from (2.7) is $76/0.6972=109$ with an estimated s.e. 10.58. Based on (2.17), the CV estimate is $\hat{\gamma}=0.61$. Hence it follows from (2.18), the proposed estimate for a continuous Model M_{th} is $109+43\,(0.61)^2/0.6972=132$ with s.e. estimate 20.09 obtained from (2.19).

4. Simulation Study

To investigate the performance of the proposed estimation procedure, we carried out a limited simulation study. The trials reported in this paper are listed in Table 1. We fixed the population size to be 100. The intensity for the ith animal is

Table 1

Description of the trials
(In each trial, $N=100$ and there are 25 animals with individual effect α_1, α_2, α_3 and α_4 respectively, $\bar{\alpha}$ = mean, CV = coefficient of variation)

trial	$\bar{\alpha}$	CV	α_1	α_2	α_3	α_4
1	.2	0	.2	.2	.2	.2
2		.2	.15	.174	.226	.25
3		.4	.1	.147	.253	.3
4		.6	.05	.12	.28	.35
5		.8	.04	.08	.235	.445
6		1.	.055	.1	.1	.545
7	.4	0	.4	.4	.4	.4
8		.2	.34	.34	.38	.54
9		.4	.28	.28	.37	.67
10		.6	.22	.22	.356	.804
11		.8	.2	.2	.24	.96
12		1.	.2	.2	.2	1.09
13	.6	0	.6	.6	.6	.6
14		.2	.45	.522	.678	.75
15		.4	.3	.441	.759	.9
16		.6	.15	.36	.84	1.05
17		.8	.12	.24	.705	1.335
18		1.	.165	.3	.3	1.636

$\lambda_i(t)=\alpha_i\beta(t)$. Two types of $\beta(t)$ were considered: $\beta(t)=1$ and $\beta(t)=t$. The population was divided into four groups of 25 animals each. The individual effect for all animals of the ith group is assumed to be α_i, $i=1-4$. These α_i's were chosen to satisfy the prespecified $\bar{\alpha}(0.2, 0.4, 0.6)$ nd CV(0. 0.2, 0.4, 0.6, 0.8, 1.0). For each fixed trial, 1000 data sets were generated for the fixed stopping time $t=3$. For each generated data, $\hat{N}_0$, $\hat{N}_1$, $\hat{N}_t$, $\hat{N}_{th}$ and $\hat{N}_{mle}$ as well as their standard error estimates were calculated. Finally these 1000 estimates and their standard errors were averaged. Based on these 1000 estimates, sample standard error as well as the sample root mean squared error (RMSE) were also obtained. Since $\hat{N}_{th}$ may yield negative estimates, we also calculated the average value of those positive ones, $\hat{N}_{th}^+$, for comparison. For each trial, the average values of D_t (the number of distinct animals captured) and C_t (the sample coverage) by the stopping time $t=3$ are also given, but the subscript t is dropped there for notational simplicity. Table 2 provides the results for $\beta(t)=1$ (Model M_h) and Table 3 for $\beta(t)=t$. All the calculations were done on a IBM RISC/6000 (model 540) work station.

Table 2

Simulation results for comparing estimates, $N=100$, stopping time $t=3$, 1000 runs, $\beta(t)=1$, * denotes the smallest RMSE, ** denotes a value $>10^6$;
$\hat{N}_0$: proposed estimator for M_t, see Eq. (2.7);
$\hat{N}_1$: proposed estimator for M_{th}, see Eq. (2.18);
$\hat{N}_t$: Becker's estimator for M_t, see Eq. (2.1);
$\hat{N}_{th}$: Becker's estimator for M_{th}, see Eq. (2.2);
$\hat{N}_{th}^+$: only positive $\hat{N}_{th}$ is considered, number in parentheses is the number of trials yielding positive values out of 1000 runs;
$\hat{N}_{mle}$: MLE under Model M_t, see Eq. (2.3).

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
1	$\hat{N}_0$	105	5	24.5	24.8	25.3
$\bar{\alpha}=.2$	$\hat{N}_1$	110	10	28.5	30.0	31.6
CV = 0	$\hat{N}_t$	104	4	23.7	23.9	24.2
$D=45$	$\hat{N}_{th}$	52	−48	−2233.2	560.6	562.7
$C=.451$	$\hat{N}_{th}^+$ (898)	125	25	814.2	210.7	212.2
	$\hat{N}_{mle}$	103	3	23.5	23.8	24.0*
2	$\hat{N}_0$	102	2	23.6	25.3	25.4
$\bar{\alpha}=.2$	$\hat{N}_1$	107	7	27.6	28.9	29.7
CV = .2	$\hat{N}_t$	101	1	22.7	24.9	24.9
$D=45$	$\hat{N}_{th}$	86	−14	324.1	542.1	542.3
$C=.460$	$\hat{N}_{th}^+$ (903)	139	39	2320.0	412.1	413.9
	$\hat{N}_{mle}$	100	0	22.5	24.7	24.7*

Table 2 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
3	$\hat{N}_0$	93	−7	20.3	21.2	22.3*
$\bar{\alpha}=.2$	$\hat{N}_1$	98	−2	24.4	24.9	25.0
CV = .4	$\hat{N}_t$	91	−9	19.3	20.8	22.7
$D=43$	$\hat{N}_{th}$	103	3	8921.6	866.3	866.3
$C=.488$	$N_{th}^+(906)$	145	45	10858.5	881.8	882.9
	$\hat{N}_{mle}$	91	−9	19.1	20.6	22.5
4	$\hat{N}_0$	81	−19	16.4	17.6	25.9*
$\bar{\alpha}=.2$	$\hat{N}_1$	86	−14	20.4	24.4	28.1
CV = .6	$\hat{N}_t$	79	−21	15.3	17.0	27.0
$D=42$	$\hat{N}_{th}$	−5	−105	**	3449.6	3451.2
$C=.538$	$N_{th}^+(885)$	152	52	56812.6	1488.1	1489.0
	$\hat{N}_{mle}$	79	−21	15.1	17.0	27.0
5	$\hat{N}_0$	70	−30	13.1	14.0	33.1
$\bar{\alpha}=.2$	$\hat{N}_1$	76	−24	17.4	19.1	30.7*
CV = .8	$\hat{N}_t$	67	−33	11.8	13.1	35.5
$D=39$	$\hat{N}_{th}$	57	−43	1566.3	511.1	512.9
$C=.587$	$N_{th}^+(872)$	120	20	3868.0	451.1	451.5
	$\hat{N}_{mle}$	67	−33	11.6	13.1	35.5
6	$\hat{N}_0$	61	−39	10.6	12.3	40.9
$\bar{\alpha}=.2$	$\hat{N}_1$	68	−32	15.7	17.8	36.6*
CV = 1.	$\hat{N}_t$	57	−43	9.0	11.5	44.5
$D=37$	$\hat{N}_{th}$	59	−41	667.9	382.1	384.3
$C=.625$	$N_{th}^+(850)$	110	10	2691.6	309.1	309.3
	$\hat{N}_{mle}$	57	−43	8.7	11.1	44.4
7	$\hat{N}_0$	101	1	10.2	9.7	9.8
$\bar{\alpha}=.4$	$\hat{N}_1$	103	3	11.9	11.4	11.8
CV = 0	$\hat{N}_t$	101	1	10.1	9.8	9.9
$D=70$	$\hat{N}_{th}$	105	5	168.6	193.5	193.6
$C=.700$	$N_{th}^+(968)$	121	21	333.6	156.0	157.4
	$\hat{N}_{mle}$	100	0	9.7	9.4	9.4*
8	$\hat{N}_0$	99	−1	9.9	10.5	10.5
$\bar{\alpha}=.4$	$\hat{N}_1$	102	2	12.0	12.5	12.7
CV = .2	$\hat{N}_t$	98	−2	9.6	10.2	10.4
$D=69$	$\hat{N}_{th}$	146	46	63120.5	2418.6	2419.0
$C=.704$	$N_{th}^+(967)$	189	89	68182.0	2412.9	2414.5
	$\hat{N}_{mle}$	98	−2	9.2	10.0	10.2*
9	$\hat{N}_0$	94	−6	9.2	9.7	11.4*
$\bar{\alpha}=.4$	$\hat{N}_1$	99	−1	12.3	12.9	12.9
CV = .4	$\hat{N}_t$	91	−9	8.6	8.8	12.6
$D=67$	$\hat{N}_{th}$	106	6	1557.0	679.5	679.5
$C=.717$	$N_{th}^+(952)$	152	52	4121.9	564.5	566.9
	$\hat{N}_{mle}$	91	−9	8.2	8.7	12.5

Table 2 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
10	$\hat{N}_0$	87	−13	8.3	9.7	16.2
$\bar{\alpha}=.4$	$\hat{N}_1$	94	−6	12.6	12.6	14.0*
CV = .6	$\hat{N}_t$	82	−18	7.3	8.7	20.0
$D=63$	$\hat{N}_{th}$	100	0	1442.9	463.1	463.1
$C=.735$	N_{th}^{+} (926)	145	45	2572.3	385.5	388.1
	$\hat{N}_{mle}$	82	−18	6.8	7.5	19.5
11	$\hat{N}_0$	79	−21	7.4	8.6	22.7
$\bar{\alpha}=.4$	$\hat{N}_1$	90	−10	13.0	12.9	16.3*
CV = .8	$\hat{N}_t$	72	−28	6.0	6.7	28.8
$D=59$	$\hat{N}_{th}$	30	−70	**	2037.6	2038.8
$C=.758$	N_{th}^{+} (870)	192	92	35239.8	1261.4	1264.8
	$\hat{N}_{mle}$	73	−27	5.5	5.7	27.6
12	$\hat{N}_0$	69	−31	6.1	8.3	32.1
$\bar{\alpha}=.4$	$\hat{N}_1$	81	−19	12.0	12.2	22.6*
CV = 1.	$\hat{N}_t$	61	−39	4.6	4.8	39.3
$D=54$	$\hat{N}_{th}$	121	21	30883.0	2134.1	2134.2
$C=.788$	N_{th}^{+} (820)	257	157	79031.4	2053.7	2059.7
	$\hat{N}_{mle}$	63	−37	4.1	7.2	37.7
13	$\hat{N}_0$	100	0	5.9	6.0	6.0
$\bar{\alpha}=.6$	$\hat{N}_1$	101	1	6.8	6.7	6.8
CV = 0	$\hat{N}_t$	100	0	6.1	6.4	6.4
$D=83$	$\hat{N}_{th}$	118	18	2794.2	579.5	579.8
$C=.835$	N_{th}^{+} (994)	128	28	3271.16	548.1	548.8
	$\hat{N}_{mle}$	100	0	5.6	5.7	5.7*
14	$\hat{N}_0$	99	−1	5.9	6.0	6.1
$\bar{\alpha}=.6$	$\hat{N}_1$	100	0	7.1	7.1	7.1
CV = .2	$\hat{N}_t$	98	−2	5.9	6.1	6.4
$D=83$	$\hat{N}_{th}$	124	24	4488.8	754.6	755.0
$C=.837$	N_{th}^{+} (992)	136	36	4977.7	725.6	726.5
	$\hat{N}_{mle}$	98	−2	5.4	5.7	6.0*
15	$\hat{N}_0$	94	−6	5.5	6.3	8.7
$\bar{\alpha}=.6$	$\hat{N}_1$	97	−3	7.5	7.8	8.4*
CV = .4	$\hat{N}_t$	91	−9	5.3	5.8	10.7
$D=79$	$\hat{N}_{th}$	95	−5	−874.8	319.2	319.2
$C=.842$	N_{th}^{+} (983)	111	11	119.0	81.7	82.4
	$\hat{N}_{mle}$	91	−9	4.7	5.0	10.3
16	$\hat{N}_0$	84	−16	4.7	5.7	17.0
$\bar{\alpha}=.6$	$\hat{N}_1$	88	−12	7.1	7.2	14.0*
CV = .6	$\hat{N}_t$	80	−20	4.3	5.7	20.8
$D=72$	$\hat{N}_{th}$	311	211	**	7080.1	7083.2
$C=.861$	N_{th}^{+} (969)	356	256	**	7161.1	7166.3
	$\hat{N}_{mle}$	81	−19	3.7	6.2	20.0

Table 2 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
17	$\hat{N}_0$	78	−22	4.4	7.7	23.3
$\bar{\alpha}=.6$	$\hat{N}_1$	84	−16	7.7	8.0	17.9*
CV = .8	$\hat{N}_t$	71	−29	3.6	2.4	29.1
$D=67$	$\hat{N}_{th}$	37	−63	**	1363.9	1365.4
$C=.871$	N^+_{th} (939)	127	27	2200.2	390.2	391.1
	$\hat{N}_{mle}$	73	−27	3.0	4.7	27.4
18	$\hat{N}_0$	76	−24	4.7	8.0	25.3
$\bar{\alpha}=.6$	$\hat{N}_1$	92	−8	11.3	11.9	14.3*
CV = 1.	$\hat{N}_t$	66	−34	3.3	7.4	34.8
$D=64$	$\hat{N}_{th}$	−147	−247	**	3850.0	3857.9
$C=.851$	N^+_{th} (810)	162	62	3556.7	396.1	400.9
	$\hat{N}_{mle}$	69	−31	2.7	6.1	31.6

Table 3

Simulation results for comparing estimates, $N=100$, stopping time $t=3$, 1000 runs, $\beta(t)=t$, * denotes the smallest RMSE, ** denotes a value $>10^6$;
$\hat{N}_0$: proposed estimator for M_t, see Eq. (2.7);
$\hat{N}_1$: proposed estimator for M_{th}, see Eq. (2.18);
$\hat{N}_t$: Becker's estimator for M_t, see Eq. (2.1);
$\hat{N}_{th}$: Becker's estimator for M_{th}, see Eq. (2.2);
$\hat{N}^+_{th}$: only positive $\hat{N}_{th}$ is considered, number in parentheses is the number of trials yielding positive values out of 1000 runs;
$\hat{N}_{mle}$: MLE under Model M_t, see Eq. (2.3).

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
1	$\hat{N}_0$	103	3	14.9	15.6	15.9
$\bar{\alpha}=.2$	$\hat{N}_1$	106	6	17.5	18.9	19.8
CV = 0	$\hat{N}_t$	102	2	14.5	15.0	15.1
$D=59$	$\hat{N}_{th}$	22	−78	**	2218.4	2219.8
$C=.593$	N^+_{th} (937)	134	34	998.8	302.1	304.0
	$\hat{N}_{mle}$	101	1	14.2	14.9	14.9*
2	$\hat{N}_0$	100	0	14.2	14.7	14.7
$\bar{\alpha}=.2$	$\hat{N}_1$	104	4	17.1	17.8	18.2
CV = .2	$\hat{N}_t$	99	−1	13.7	114.2	14.2
$D=59$	$\hat{N}_{th}$	84	−16	−87.1	295.0	295.4
$C=.603$	N^+_{th} (939)	124	24	571.0	187.9	189.4
	$\hat{N}_{mle}$	99	−1	13.4	13.9	13.9*
3	$\hat{N}_0$	92	−8	12.6	12.2	14.6*
$\bar{\alpha}=.2$	$\hat{N}_1$	97	−3	15.9	15.8	16.1
CV = .4	$\hat{N}_t$	90	−10	11.9	11.8	15.5
$D=57$	$\hat{N}_{th}$	−86	−186	**	6026.1	6029.0
$C=.628$	N^+_{th} (938)	144	44	4010.7	519.2	521.1
	$\hat{N}_{mle}$	90	−10	11.5	11.7	15.4

Table 3 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
4	$\hat{N}_0$	80	−20	10.0	9.6	22.2
$\bar{\alpha}=.2$	$\hat{N}_1$	85	−15	13.2	14.0	20.5*
CV = .6	$\hat{N}_t$	78	−22	9.3	10.3	24.3
$D=53$	$\hat{N}_{th}$	86	−14	143.3	335.6	335.9
$C=.676$	$N_{th}^{+}(940)$	119	19	986.86	251.7	252.4
	$\hat{N}_{mle}$	78	−22	9.0	10.1	24.2
5	$\hat{N}_0$	71	−29	8.3	9.4	30.5
$\bar{\alpha}=.2$	$\hat{N}_1$	77	−23	12.0	13.6	26.7*
CV = .8	$\hat{N}_t$	68	−32	7.4	10.2	33.6
$D=50$	$\hat{N}_{th}$	91	−9	45296.1	1547.6	1547.6
$C=.712$	$N_{th}^{+}(904)$	172	72	58835.2	1495.6	1497.3
	$\hat{N}_{mle}$	68	−32	7.0	9.6	33.4
6	$\hat{N}_0$	64	−36	7.4	7.6	36.8
$\bar{\alpha}=.2$	$\hat{N}_1$	73	−27	12.5	12.6	29.8*
CV = 1.	$\hat{N}_t$	59	−41	5.9	9.1	42.0
$D=46$	$\hat{N}_{th}$	46	−54	−4420.4	629.4	631.7
$C=.728$	$N_{th}^{+}(831)$	119	19	2035.0	289.2	289.8
	$\hat{N}_{mle}$	59	−41	5.5	5.0	41.3
7	$\hat{N}_0$	100	0	5.9	6.0	6.0
$\bar{\alpha}=.4$	$\hat{N}_1$	101	1	6.8	6.7	6.8
CV = 0	$\hat{N}_t$	100	0	6.1	6.4	6.4
$D=83$	$\hat{N}_{th}$	118	18	2794.2	579.5	579.8
$C=.835$	$N_{th}^{+}(994)$	128	28	3271.1	548.1	548.8
	$\hat{N}_{mle}$	100	0	5.6	5.7	5.7*
8	$\hat{N}_0$	99	−1	5.9	6.1	6.2
$\bar{\alpha}=.4$	$\hat{N}_1$	101	1	7.2	7.4	7.5
CV = .2	$\hat{N}_t$	98	−2	6.0	6.3	6.6
$D=83$	$\hat{N}_{th}$	118	18	677.1	239.8	240.5
$C=.837$	$N_{th}^{+}(994)$	120	20	690.0	238.1	238.9
	$\hat{N}_{mle}$	98	−2	5.4	5.7	6.0*
9	$\hat{N}_0$	96	−4	5.8	6.0	7.2*
$\bar{\alpha}=.4$	$\hat{N}_1$	100	0	8.2	8.0	8.0
CV = .4	$\hat{N}_t$	93	−7	5.5	6.4	9.5
$D=80$	$\hat{N}_{th}$	92	−8	2633.9	1635.6	1635.6
$C=.837$	$N_{th}^{+}(982)$	152	52	16133.8	1101.0	1102.3
	$\hat{N}_{mle}$	93	−7	4.9	5.2	8.7
10	$\hat{N}_0$	91	−9	5.5	7.0	11.4
$\bar{\alpha}=.4$	$\hat{N}_1$	99	−1	9.3	9.4	9.5*
CV = .6	$\hat{N}_t$	85	−15	4.8	6.9	16.5
$D=76$	$\hat{N}_{th}$	174	74	34503.1	1660.2	1661.8
$C=.840$	$N_{th}^{+}(955)$	202	102	36430.0	1690.2	1693.3
	$\hat{N}_{mle}$	86	−14	4.2	5.4	15.0

Table 3 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
11	$\hat{N}_0$	85	−15	5.4	6.9	16.5
$\bar{\alpha}=.4$	$\hat{N}_1$	99	−1	11.0	11.2	11.2*
CV=.8	$\hat{N}_t$	76	−24	4.1	5.8	24.7
$D=71$	$\hat{N}_{th}$	−1157	−1257	**	37681.4	37702.4
$C=.840$	$N_{th}^{+}(874)$	173	73	4998.9	609.3	613.7
	$\hat{N}_{mle}$	79	−21	3.5	6.9	22.1
12	$\hat{N}_0$	76	−24	4.7	8.0	25.3
$\bar{\alpha}=.4$	$\hat{N}_1$	92	−8	11.3	11.9	14.3*
CV=1.	$\hat{N}_t$	66	−34	3.3	7.4	34.8
$D=64$	$\hat{N}_{th}$	−147	−247	**	3850.0	3857.9
$C=.851$	$N_{th}^{+}(810)$	162	62	3556.7	396.1	400.9
	$\hat{N}_{mle}$	69	−31	2.7	6.1	31.6
13	$\hat{N}_0$	100	0	3.2	3.3	3.3
$\bar{\alpha}=.6$	$\hat{N}_1$	101	1	3.6	3.6	3.7
CV=0	$\hat{N}_t$	100	0	3.7	3.7	3.7
$D=93$	$\hat{N}_{th}$	104	4	39.7	43.5	43.7
$C=.932$	$N_{th}^{+}(10^3)$	104	4	39.7	43.5	43.7
	$\hat{N}_{mle}$	100	0	3.0	3.0	3.0*
14	$\hat{N}_0$	99	−1	3.3	3.4	3.5*
$\bar{\alpha}=.6$	$\hat{N}_1$	100	0	3.9	4.0	4.0
CV=.2	$\hat{N}_t$	98	−2	3.7	3.6	4.1
$D=92$	$\hat{N}_{th}$	−38	−138	**	4405.3	4407.5
$C=.930$	$N_{th}^{+}(995)$	112	12	1036.5	307.6	307.8
	$\hat{N}_{mle}$	98	−2	2.9	3.0	3.6
15	$\hat{N}_0$	96	−4	3.4	3.6	5.4
$\bar{\alpha}=.6$	$\hat{N}_1$	99	−1	4.8	4.9	5.0*
CV=.4	$\hat{N}_t$	93	−7	3.4	4.5	8.3
$D=89$	$\hat{N}_{th}$	107	7	613.8	255.7	255.8
$C=.925$	$N_{th}^{+}(991)$	112	12	650.3	249.9	250.2
	$\hat{N}_{mle}$	94	−6	2.6	4.0	7.2
16	$\hat{N}_0$	88	−12	3.2	4.2	12.7
$\bar{\alpha}=.6$	$\hat{N}_1$	92	−8	5.2	5.3	9.6*
CV=.6	$\hat{N}_t$	83	−17	2.8	5.3	17.8
$D=82$	$\hat{N}_{th}$	121	21	10079.4	1032.3	1032.5
$C=.926$	$N_{th}^{+}(978)$	142	42	11207.3	1001.7	1002.6
	$\hat{N}_{mle}$	85	−15	2.1	5.3	15.9
17	$\hat{N}_0$	82	−18	3.1	2.7	18.2
$\bar{\alpha}=.6$	$\hat{N}_1$	90	−10	6.4	6.6	12.0*
CV=.8	$\hat{N}_t$	75	−25	2.5	5.0	25.5
$D=76$	$\hat{N}_{th}$	53	−47	−2968.2	540.6	542.6
$C=.923$	$N_{th}^{+}(929)$	118	18	853.4	201.6	202.4
	$\hat{N}_{mle}$	78	−22	1.7	3.6	22.3

Table 3 (continued)

trial	method	estimate	bias	estimated s.e.	sample s.e.	sample RMSE
18	$\hat{N}_0$	84	−16	3.7	6.6	17.3
$\bar{\alpha}=.6$	$\hat{N}_1$	103	3	10.8	10.1	10.5*
CV = 1.	$\hat{N}_t$	72	−28	2.4	7.2	28.9
$D=71$	$\hat{N}_{th}$	66	−34	−914.6	568.0	569.0
$C=.902$	N_{th}^{+}(855)	162	62	2807.9	304.3	399.1
	$\hat{N}_{mle}$	77	−23	1.7	3.7	23.3

The numerical results indicate that when CV = 0 (Model M_t), the MLE performs the best in the sense of the smallest bias and RMSE. When CV = 0.2, the MLE still outperforms the others. When CV = 0.4, the MLE, proposed $\hat{N}_0$ and Becker $\hat{N}_t$ are comparable (note these three estimators derived for Model M_t have similar performance in most trials). When CV is relatively large (CV > 0.4 generally), the proposed $\hat{N}_1$ generally produces the smallest bias and RMSE.

The standard error estimate for $\hat{N}_0$ and $\hat{N}_1$ derived in (2.19) given in column 5 of each table are generally satisfactory compared with the sample standard error in column 6.

Notice that MSE is the sum of variance and the squared of bias. For Model M_{th}, our estimator $\hat{N}_1$ when compared with estimators for Model M_t generally reduces bias but increases the variance due to estimation of CV. Also it is obvious that we need sufficiently abundant data to estimate CV. Thus whether the reduction in squared of bias can compensate for the increase in variance clearly depends on the value of CV and the abundance of data (i.e. $\bar{\alpha}$ and $\beta(t)$). When CV is relatively small, the usual estimators without estimating CV are not seriously biased, so the improvement in bias is quite limited. Thus our method can not effectively reduce MSE. Therefore there seems no advantage in estimating CV in these situations. However, if CV is relatively large, the usual estimators have a quite large bias, it then warrants the use of the proposed procedure to reduce MSE as long as the data are not sparse. However, the general guidelines about how abundant data we need is still unclear to us.

In all tables, the estimator $\hat{N}_{th}$ apparently works unsatisfactorily. Since extremely large or negative estimates can result, as also found by ANDERSON and WILSON (1989), $\hat{N}_{th}$ generally produces the largest bias and RMSE. The s.e. estimate provided in BECKER (1984) differs greatly with the sample values.

In conclusion, if the degree of heterogeneity is not large (CV $\leqq$ 0.4), traditional estimator without considering heterogeneity is still appropriate. When the degree of heterogeneity is relatively large (CV > 0.4) and there are sufficient data to generate a stable estimator of CV, our proposed estimation procedure incorporating CV term is thus recommended.

Acknowledgement

The authors thank Professor Kenneth Burnham for making the paper of ANDERSON and WILSON (1989) available. Research was supported by the National Science Council under contracts NSC-79-0208-M007-18 and 80-0208-M007-39.

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Received Aug. 1991

ANNE CHAO
Institute of Statistics
National Tsing Hua University
Hsin Chu, Taiwan 30043

S. M. LEE
Department of Statistics
Feng-Chia University
Tai-Chung, Taiwan 40724